Instructions:

- Please read questions carefully
- Show work/reasoning for (partial) credit
- Conduct all hypothesis tests at significance level $\alpha = 0.05$
- Note point values on questions
- Make sure your exam has 4 questions

1. / 28
2. / 28
3. / 28
4. / 16

/ 100
1. (28 pts.) Industries such as refineries, steel mills, and food processing plants release wastewater into the environment. This wastewater should be neutralized before it is released. Treated wastewater pH values are recorded for a sample from an industry using a particular treatment process:

\[ 6.2, 6.5, 7.6, 7.7, 7.0, 7.0, 7.2, 6.8, 7.5, 8.1, 7.1, 7.0, 7.1, 7.8, 8.5 \]

(There are 15 observations with a sample average of 7.273 and a sample standard deviation of 0.602.)

Based upon these data, is there evidence that the treatment process does not yield an average pH of 7 as desired? Conduct the appropriate test by answering the questions below:

a. State the null and alternative hypotheses for this test

\[ H_0 : \mu = 7 \]
\[ H_a : \mu \neq 7 \]

where \( \mu \) is the true mean treated wastewater pH.

b. Draw the density curve the test statistic follows if the null hypothesis is true. Correctly add, along the horizontal axis, the labels "Accept \( H_0 \)" and "Reject \( H_0 \)" regions. Label the horizontal axis with the numerical value(s) that separate the accept and reject regions.

\[ \text{Area} > \frac{\alpha}{2} = .025 \]
\[ t_{14} \]
\[ \text{Area} > \frac{\alpha}{2} = .025 \]
\[ \text{REJECT } H_0 \]
\[ -2.145 \]
\[ 2.145 \]
\[ \text{ACCEPT } H_0 \]
\[ \text{REJECT } H_0 \]

C. Compute the test statistic (it's either a \( t \) or a \( z \)).

\[ t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{7.273 - 7}{0.602/\sqrt{15}} \approx 1.76 \]

d. Should the null hypothesis be accepted or rejected?

\[ \text{1 Gray, David and Marshall, Jeff (1992), "How to choose a pH measurement system"}, \text{ Pollution Engineering}, \text{ pp. 45-47.} \]
e. Give a 95% confidence interval for $\mu$.

$$\bar{x} \pm 2.145 \frac{s}{\sqrt{n}} = 7.273 \pm 2.145 \frac{0.602}{\sqrt{15}} = (6.94, 7.61)$$

f. What must we assume about the population of treated wastewater pH reading values . . .

i. . . . in the hypothesis test conducted above (if anything)?

Follows some normal $(n < 40)$

ii. . . . in the confidence interval constructed above (if anything)?

Follows some normal $(n < 40)$
2. (28 pts) The package of GE 60 Watt light bulbs I have at home claims they last 1250 hours (on average). Suppose I take a sample of 50 light bulbs and they last an average of 1230 hours with standard deviation of 80 hours.

Is the true mean duration of the bulbs less than 1250 hours? Conduct the appropriate test by answering the questions below:

a. State the null and alternative hypotheses for this test

$$H_0: \mu = 1250$$
$$H_a: \mu < 1250$$

where $\mu$ is the true mean lifetime.

b. Draw the density curve the test statistic follows if the null hypothesis is true. Correctly add, along the horizontal axis, the labels "Accept $H_0$" and "Reject $H_0$" regions. Label the horizontal axis with the numerical value(s) that separate the accept and reject regions.

\[ \text{Area} = P(Z < -1.645) \]

\[ \text{REJECT } H_0 \]

\[ \text{ACCEPT } H_0 \]

\[ Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1230 - 1250}{80/\sqrt{50}} = -1.77 \]

d. Should the null hypothesis be accepted or rejected?
e. Give a 95% confidence interval for $\mu$.

$$\bar{x} \pm 1.96 \frac{s}{\sqrt{n}} = 1230 \pm 1.96 \frac{80}{\sqrt{50}} = (1207.8, 1252.2)$$

f. What must we assume about the population of light bulb lifetimes . . .

i. . . . in the hypothesis test conducted above (if anything)?

$$\text{NOTHING (} n \geq 40 \text{)}$$

ii. . . . in the confidence interval constructed above (if anything)?

$$\text{NOTHING (} n \geq 90 \text{)}$$
3. (28 pts.) Devor, Chang, and Sutherland (1992) examined a process for manufacturing electrical resistors that have a nominal resistance of 100 ohms with a specification of ±2 ohms. Suppose management has expressed a concern that the true proportion of resistors with resistances outside the specifications has increased from the historical level of 10%. A random sample of 180 resistors yielded 46 with resistances outside the specifications.

Conduct the appropriate test by answering the questions below:

a. State the null and alternative hypotheses for this test

\[ H_0: \quad \rho = 0.10 \]

\[ H_a: \quad \rho \neq 0.10 \]

in terms of \( \rho \), the proportion of resistors that don't meet the specification.

b. Draw the density curve the test statistic follows if the null hypothesis is true. Correctly add, along the horizontal axis, the labels "Accept \( H_0 \)" and "Reject \( H_0 \)" regions. Label the horizontal axis with the numerical value(s) that separate the accept and reject regions.

\[ \text{Area} = \alpha = 0.05 \]

[Diagram showing normal distribution with critical values]

Accept \( H_0 \)

Reject \( H_0 \)

1.645

\[ z = \frac{\hat{p} - \rho}{\sqrt{\frac{\rho(1-\rho)}{n}}} = \frac{46/180 - 0.1}{\sqrt{(0.1)(0.9)/180}} \approx 1.96 \]

d. Should the null hypothesis be accepted or rejected?

\[ \approx C.96 \]

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e. Give an approximate 95% confidence interval for $p$.

\[ \hat{p} \pm 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = (0.192, 0.319) \]

f. What must we assume about the population of resistor resistance values . . .

i. . . . in the hypothesis test conducted above (if anything)?

\[
\text{NOTHING} \quad (n \geq 30)
\]

ii. . . . in the confidence interval constructed above (if anything)?

\[
\text{NOTHING} \quad (n \geq 30)
\]
4. (16 pts.) Method of Moments

a. If a particle moves in a plane so that its horizontal and vertical distances from the origin are each, independently, normal with mean zero with the same variance $\theta^2$, then the Rayleigh density is the density of the particle's distance from the origin. The Rayleigh density, given by

$$f(x) = \begin{cases} \frac{x}{\theta^2} e^{-x^2/(2\theta^2)} & x \geq 0 \\ 0 & x < 0 \end{cases},$$

has mean $\mu = \frac{\sqrt{2\pi}}{\theta}$. Suppose we observe 20 distances from the origin for such a particle with an average of 24.11. Determine the method of moments estimate of $\theta$.

**Setting**

$\mu = \bar{x}$

**We have**

$$\sqrt{\frac{\pi}{2}} \theta = \bar{x}$$

$$\hat{\theta}_{MM} = \frac{\bar{x}}{\sqrt{\frac{\pi}{2}}} \approx \frac{24.11}{1.2533} = 19.24$$

b. Suppose runners in a race are wearing numbers ("bibs") running from 1 through $N$, with $N$ unknown. For such a population of numbers it can be readily shown that $\mu = \frac{N+1}{2}$. You watch this race seeing just the following numbers:

126, 33, 213, 172, 289

Give the method of moments estimate of the number of runners, $N$.

**Setting**

$\mu = \bar{x}$

**We have**

$$\frac{N+1}{2} = 166.6$$

$$\hat{N}_{MM} = 2(166.6) - 1 = 332.2$$