1. (18 pts.) In 1912 the Titanic sank without enough lifeboats for the passengers and crew. The table below looks at survival by gender.

**Titanic Survival by Gender**

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survived</td>
<td>367</td>
<td>344</td>
<td>711</td>
</tr>
<tr>
<td>Died</td>
<td>1364</td>
<td>126</td>
<td>1490</td>
</tr>
<tr>
<td>Total</td>
<td>1731</td>
<td>470</td>
<td>2201</td>
</tr>
</tbody>
</table>

In working this problem, use the notation

\[ S = \text{Survived the sinking} \]
\[ D = \text{did not survive the sinking (Died)} \]
\[ M = \text{Male} \]
\[ F = \text{Female} \]

a. What proportion of the passengers were women? For full-credit, use correct probability notation when writing out your answer (i.e. write either P(A|B) or P(A) with A, B appropriately replaced by event(s) listed above).

\[ P(F) = \frac{470}{2201} \approx .214 \]

b. What proportion of women survived the sinking of the Titanic? For full-credit, use correct probability notation when writing out your answer (i.e. write either P(A|B) or P(A) with A, B appropriately replaced by event(s) listed above).

\[ P(S|F) = \frac{344}{470} \approx .732 \]

c. Given that a passenger survived, what is the chance that they were female? For full-credit, use correct probability notation when writing out your answer (i.e. write either P(A|B) or P(A) with A, B appropriately replaced by event(s) listed above).

\[ P(F|S) = \frac{344}{711} \approx .484 \]
2. (12 pts.) *Keno* is a lottery game. Here are the details of a *Keno* bet you can make in Deadwood, South Dakota via video machine:

You select 4 different numbers from 1 through 80 inclusive. There will be 20 numbers chosen as winners by the house, with the other 60 losers. Net winnings for a $1 bet depend on how many of the 20 winning numbers you match. In particular, here are your winnings for the various possible numbers of matches:

<table>
<thead>
<tr>
<th>Winning Numbers Matched</th>
<th>Net Winnings (Dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
</tr>
</tbody>
</table>

- **a.** What is the chance, in a single $1 bet, that you win $90?

\[
P(\text{Win $90}) = P(\text{Match 4}) = \frac{\binom{20}{4} \binom{60}{0}}{\binom{80}{4}}
\]

- **b.** What is the chance, in a single $1 bet, that you lose a dollar?

\[
P(\text{Lose $1}) = P(\text{Match 0 or Match 1}) = P(\text{Match 0}) + P(\text{Match 1})
\]

\[
= \frac{\binom{20}{0} \binom{60}{4}}{\binom{80}{4}} + \frac{\binom{20}{1} \binom{60}{3}}{\binom{80}{4}}
\]
3. (12 pts.) Multiple Choice (circle the correct response, no work is needed)

a. If each of the three pairs of events

\[ A \text{ and } B, \ A \text{ and } C, \ & \ B \text{ and } C \]

have no outcomes in common, then we say

i. \underline{A, B, C are disjoint events}
ii. \underline{A, B, C are independent events}
iii. \underline{A, B, C are complimentary events}

b. If

\[ P(A \text{ and } B \text{ and } C) = P(A)P(B)P(C) \]
\[ P(A \text{ and } B) = P(A)P(B) \]
\[ P(A \text{ and } C) = P(A)P(C) \]
\[ P(B \text{ and } C) = P(B)P(C) \]

then we say

i. \underline{A, B, C are disjoint events}
ii. \underline{A, B, C are independent events}
iii. \underline{A, B, C are complimentary events}

c. \[ P(A) = P(A \mid B)P(B) + P(A \mid B')P(B') \] is

i. \underline{Always true}
ii. Sometimes true
iii. Never true

d. \[ P(A \text{ or } B) = P(A) + P(B) \] is

i. \underline{Always true}
ii. \underline{Sometimes true}
iii. Never true
4. (24 pts.) A system consists of two components. The probability that the first component works during its design life is 0.90, the probability that at least one of the two components does so is 0.95, and the probability that both components do so is 0.80. (Suggestion: Let $F =$ first component works during its design life, $S =$ second component works during its design life, and draw a Venn-diagram.)

![Venn Diagram]

a. What is the probability that the second component works during its design life?

$$P(S) = .85$$

b. Using the definition of independence, show why the two components do not work independently.

$$0.80 = P(F \text{ and } S) = P(F)P(S) = (.90)(.85) = .765$$

c. What is the chance that exactly one component works (i.e. just the first component works or just the second component works)?

$$P((F \text{ and } S^c) \text{ or } (F^c \text{ and } S)) = .15$$

d. Given the first component works throughout its design life, what is the chance the second one does so?

$$P(S|F) = \frac{P(S \cap F)}{P(F)} = \frac{0.80}{0.90} = \frac{8}{9} = .89$$
5. (9 pts.) A system of \( n \) components is called a \( k \)-out-of-\( n \) system provided it functions if and only if at least \( k \) of its components function. (Example: Perhaps a roof being supported by \( n \) beams remains intact as long as at least some number of the beams are of a threshold level of strength.)

a. A 1-out-of-\( n \) system is better known as a \( \boxed{\text{parallel}} \) \( \boxed{\text{series}} \) (circle one) system.

b. A \( n \)-out-of-\( n \) system is better known as a \( \boxed{\text{parallel}} \) \( \boxed{\text{series}} \) (circle one) system.

c. Suppose that the components in a \( 8 \)-out-of-\( 10 \) system work independently with chance \( p = 0.9 \). Write, but don't evaluate, an expression for the chance the system works.

\[
\binom{10}{8} (0.9)^8 (0.1)^2 + \binom{10}{9} (0.9)^9 (0.1)^1 + \binom{10}{10} (0.9)^{10} (0.1)^0
\]

6. (12 pts.) This problem concerns the tossing of a pair of four-sided (‘tetrahedral’) dice (each has the equally-likely numbers 1,2,3,4).

a. If the pair are tossed once, what is the chance of a sum of 3?

\[
P(\text{sum = 3}) = \frac{2}{16} = 0.125
\]

b. What is the chance of at least one double 4 in 10 tosses of the pair of dice?

\[
P(\text{at least one double 4}) = 1 - P(\text{no double 4})
\]

\[
= 1 - \left( \frac{15}{16} \right)^{10}
\]
7. (12 pts.) In many popular board games (e.g. Trouble, Parchessi) one cannot move their players from "home" to the general playing area until they first throw a particular value on the (single) die that is thrown. Suppose that a ‘6’ must be thrown, using a fair six-sided die, to bring a player out.

a. What is the chance that you are first able to bring out a player from home to the general playing area on your fifth turn?

\[
\left( \frac{5}{6} \right)^5 \left( \frac{1}{6} \right)
\]

b. What is the chance that it takes you at least three turns to bring out a player from home to the general playing area? For full-credit, give me a numeric answer.

\[
P(\text{at least 3 turns}) = 1 - P(1 \text{ turn or 2 turns})
\]

\[
dis. \quad 1 - \left[ P(1 \text{ turn}) + P(2 \text{ turns}) \right]
\]

\[
\downarrow
\]

\[
1 - \left( \frac{1}{6} + \frac{5}{6} \cdot \frac{1}{6} \right) = \frac{25}{36}
\]

8. (6 pts.) How many passwords of length 5 can be made where each character is either a lower case letter (‘a’, ‘b’, ..., ‘z’) or a digit (‘0’, ‘1’, ..., ‘9’)?

\[
26 \text{ letters} \quad \times \quad 10 \text{ digits}
\]

\[
36^5
\]